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A revised model for the cost of quality

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Abstract *Traditionally, process improvement is considered a defect prevention effort. Current cost models consider the coupled effect of both prevention and appraisal costs on the cost of failure. This paper proposes a new model for the cost of quality, which captures the value of continuous process improvement in achieving economic operation. The model is developed to incorporate two cost functions. The first accounts for quality related costs incurred while maintaining a stable level of operation, while the second accounts for the cost of process improvement. Using incremental economics, the two cost functions are assembled and an economic criterion for evaluating improvement alternatives is developed. Numerical examples are used to illustrate potential applications and performance of the model.*

1. Introduction

Several ways of defining quality costs have been developed in the literature (Dale and Plunkett, 1999). Feigenbaum (1991) categorized operating quality costs into two major components: cost of control and cost of failure of control. The former includes appraisal and prevention costs, whereas the latter includes internal as well as external failure costs. Examples of typical cost elements under each category can be found in Juran and Godfrey (1999), and Feigenbaum (1991). Harrington (1987) has compiled a list of typical cost elements, identifying 101 prevention costs, 73 appraisal costs, 139 internal failure costs and 50 external failure costs.

In general, the objectives of identifying these costs have been to provide a scoreboard for cost control and identify opportunities for improvement. Quality experts argue that a typical company can save more money by halving poor quality costs than by doubling sales (Harrington, 1987). Gryna (1988, 2001) presented two conceptual models for the cost of conformance. Each model shows three curves: failure, prevention plus appraisal and total cost. As Gryna pointed out, the first model, depicted in Figure 1, represents the conditions that prevailed during much of the twentieth century. A major aspect of this model is the infinite costs required to attain perfection. Figure 2 represents what has been termed the right costs or par value model. The optimal quality has been shifted to the 100 percent conformance level. In

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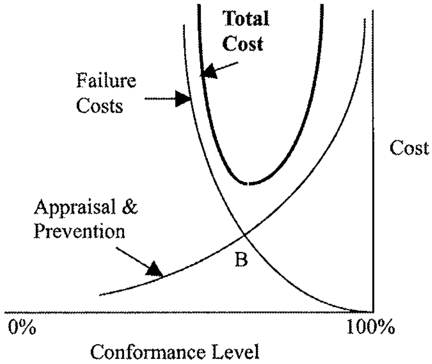

Figure 1.
The traditional model

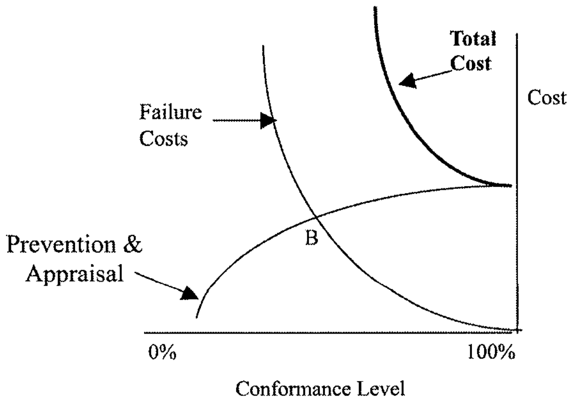

Figure 2.
The par value model

contrast to the older model, the total cost curve indicates that higher conformance costs less. However, the tendency to combine appraisal, and prevention costs in both models has been questioned by Diallo *et al.* (1995), Fine (1986) and Fine and Porteous (1988). Hwang and Aspinwall (1996) indicated that there are many arguments about the economic relationship between conformance expenditure and quality improvement, without any empirical studies to substantiate them.

During a study of the collection and use of quality-related costs Plunkett and Dale (1988) found wide differences between the models and real data. They concluded that the models are inaccurate and misleading. They presented serious doubts of the concept of the optimal quality level corresponding to a minimum point on the total cost curve.

In addition, these authors believe that both models are subject to two serious limitations. First, while the models address the relationship between process conformance level and quality-related costs they assume perfect design quality. This is indicated by a diminishing failure costs at the zero defect level. In fact, this is what would be expected for internal, but not external failure costs (e.g.

product recall, complaint adjustment, warranty replacement and the like). The later are also attributable to design quality (e.g. product reliability and durability) as well as the conditions under which the product is used. Second, both models indicate levels of conformance at which prevention and appraisal expenditures exceed failure costs. These levels appear to the right of the point of intersection of the two cost curves (see point B, Figures 1 and 2). This communicates the wrong message to top management. It is a clear statement that failures can cost less than prevention, which offers no help in justifying process improvement projects.

Ittner (1996) examined the hypothesis that conformance expenditures must continue to be increased to achieve ongoing reductions in nonconformance costs. Based on a time series analysis of quality costs reported by 49 manufacturing units of 21 companies, he observed that nonconformance cost reductions could be achieved with little or no subsequent increase in conformance expenditures. He pointed out that a micro-level examination of quality cost behavior could provide a better understanding of the underlying economics of quality improvements.

In this paper, we propose a revised model of the cost of quality. The model is developed to overcome the limitations cited above and account for the value of process improvement in achieving economic operations. In the following section, we provide a general description of the model and propose an economic criterion for evaluating process improvement projects. Section 2, provides a description of the process model considered and the assumptions made regarding the sampling procedures implemented. Sections 3 and 4 include mathematical developments of the pertaining cost functions. Detailed derivations of these functions are included in the Appendix. In Section 5, we represent an application to illustrate the performance of the proposed model. A sensitivity analysis is performed using factorial experimental design in Section 6, followed by concluding remarks in Section 7.

2. The general model

In this paper, the total cost of conformance is made to include two functions. The first estimates the costs incurred while maintaining stable operation at an existing level of conformance (L_0), and hence termed reactive costs. These include:

- the cost of monitoring the state of operation (C_m);
- the cost of inspecting production units(C_i); and
- the cost of deviating from performance targets (C_d).

In terms of the average (expected) values, the total reactive cost per unit time of operation is expressed as:

$$E(\mathrm{RC})_L = E(C_m) + E(C_i) + E(C_d).$$

The second function accounts for the cost of attaining an improved level of conformance (L_1). This entails the introduction of planned changes to the process. Consequently, the costs incurred are termed proactive costs. The two elements considered are:

(1) test and evaluation costs; and

(2) implementation costs.

In Section 5, we develop this cost function and show that the costs incurred depend on the manner by which process changes are made, their magnitudes, and the ability to evaluate resulting effects. The economic justification of these proactive expenditures can be based on the expected reduction in reactive costs. To illustrate, we consider two stable levels of conformance, the current level L_0, and an improved level L_1. We will denote the average reactive cost at each level by $E(\text{RC})_0$, and $E(\text{RC})_1$ respectively, where $E(\text{RC})_0 > E(\text{RC})_1$. This condition holds true as long as the two estimates are related to the same conformance targets. If production is expected to continue over an infinite time horizon, cost savings are best represented in terms of the capitalized equivalent amount (Thuesen and Fabrycky, 1993). Let i be the effective interest rate per period, and γ the total operation time per interest period, then the cost saving expected upon achieving the improved level L_1, is equivalent to:

$$\frac{\gamma}{i}\{E(\text{RC})_0 - E(\text{RC})_1\}.$$

On the other hand, since proactive costs usually require immediate allocation of funds, they are considered as investments made at the present time. As such, the economics of process improvement can be assessed based on the net present worth (NPW) given by:

$$\text{NPW} = \frac{\gamma}{i}\{E(\text{RC})_0 - E(\text{RC})_1\} - E(\text{IC}). \tag{1}$$

Whereas, in applications involving a finite number of production periods J, the net present worth of improvement can be obtained as:

$$\text{NPW} = \omega\gamma\{E(\text{RC})_0 - E(\text{RC})_1\} - E(\text{IC}) \tag{2}$$

where, ω is a discount factor given by:

$$\omega = \frac{1-(1+i)^{-J}}{i}. \tag{3}$$

The economic criterion would be to improve the process if the expected proactive costs do not exceed the expected net savings, in other words if $\text{NPW} > 0$. The inequality used implies that when the expected reactive costs are equal to the expected net savings, the improvement option would still be economically desirable at the specified interest i. When this criterion is not satisfied, alternative projects need to be considered. The long-term implementation of this economic criterion would lead to the new proposed model in Figure 3.

As shown above, the two curves at the bottom represent the reactive and proactive costs as a function of the process conformance level. The differences between these costs represent net savings as illustrated by the third curve. It is worth noting here that the two curves do not intersect as in traditional models. Instead, the optimal level of conformance is shown at the design target where the net worth curve approaches zero. At this point residual costs are attributable to the gap between design quality and customer expectations. The loss due to any misconception of customer expectations in specifying design targets (quantity, delivery schedules, as well as dimensions) can be substantial and should not be combined with other cost elements. Here, improvement projects should be directed towards identifying new target values to better achieve customer expectations. The incurred expenditures need to be justified based on revenue-side quality benefits (e.g., increased market share and profit margins) as well as anticipated reductions in external failure costs. Improvements of this type are usually undertaken at the system level and hence will not be addressed in the forthcoming developments.

3. Process model and cycle time

The process considered is one of discrete manufacturing of a single quality characteristic x. It is assumed that the process is essentially repetitive and that x is normally distributed with design requirements $m \pm \Delta$. A Shewhart-type measurement chart is being used to monitor the process over time. Further, we assume that a state of statistical control has been

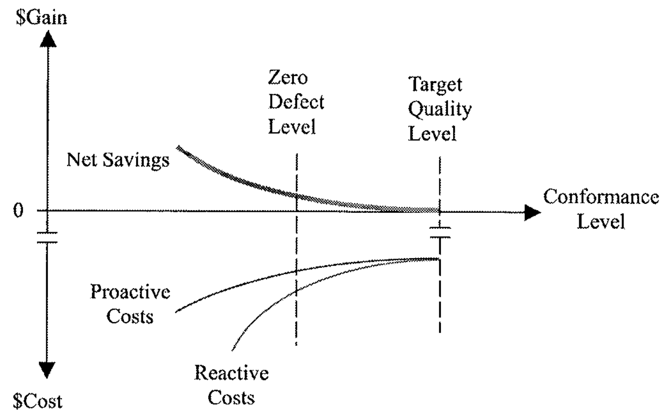

Figure 3.
Conceptual form of the proposed model

established for a period of time long enough to provide reliable estimates of the various time and cost parameters. The charting scheme consists of drawing samples of n items, every h hours and plotting the calculated statistic(s) against control limits placed k-standard deviation units around the centerline. The probability that a sample point falls outside these control limits when no change is made to the process is α, whereas the probability that a sample point falls within the control limits when a change has actually been made is β. A schematic presentation of the process model is shown in Figure 4.

As the process continues to be in control, formulated lots of N items are released to an inspection station for appraisal. The inspection station receives these lots upon an indication of the "no action" signal from the control chart. Lot acceptance is based on inspecting a random sample of size n_s. We denote the probability of acceptance by Q, and that of rejection by P. Accepted lots are passed to further processing stages, while rejected lots are routed to screening and nonconforming units are replaced. Non-destructive testing is assumed in both sampling and screening activities. Since it is assumed that nonconforming units detected are replaced, the lot size is not reduced from N. The main effect of rejecting the process or the lot is to increase the lot completion time λ_t. Delay costs are incurred when the lot completion time exceeds its processing time λ.

Now, if we assume that the time to measure a unit e is the same as that required to inspect or screen the unit and λ_r is the time to replace a nonconforming unit, then the time required to inspect a sample of n_s units and replace nonconforming units is, $\lambda_s = n_s(e + \lambda_r p)$, where p is the estimated proportion of nonconforming units. Similarly, the time required to inspect the sample, screen the lot and replace nonconforming units is, $\lambda_{sc} = N(e + \lambda_r p)$. Due to the independence between the control action limits and those of the

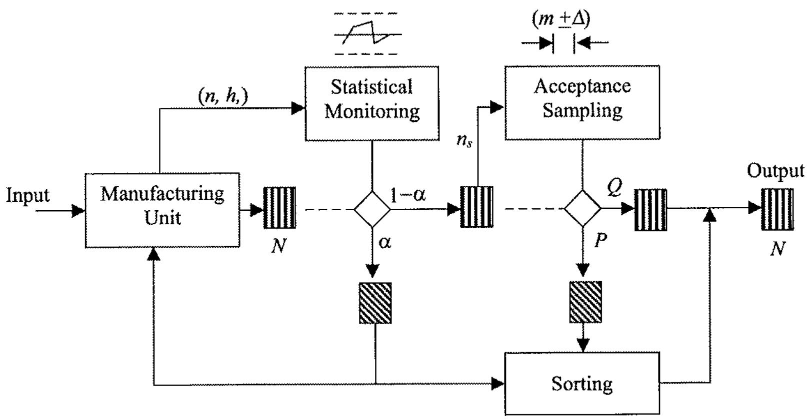

Figure 4.
Schematic presentation of the process model

product acceptance plan, the average delay that would be experienced in having N units available to the customer is (see Appendix, A1.1):

$$E(d) = (e + \lambda_r p)\left[N - Q(1-\alpha)(N - n_s)\right] \quad (4)$$

with an associated mean squared deviation in the form:

$$\nu^2 = (e + \lambda_r p)^2\left[N^2 - Q(1-\alpha)\left(N^2 - n_s^2\right)\right]. \quad (5)$$

Based on equation (4) above, it appears that the expected delay reaches a minimum when the term starting with Q is at maximum. Since the probability of acceptance Q is a decreasing function of p under any sampling plan, the expected delay is always an increasing function of p. The function will reach a minimum at $p = 0$ where $Q = 1$. A maximum average delay will occur at $n_s = N$.

4. Reactive cost of quality

In this section we formulate a model for estimating quality related costs at a stable level characterized by (μ, σ^2) the process average and variance. These include process monitoring and product inspection costs, as well as the loss due to deviation from performance targets.

4.1 Process monitoring cost

This element includes the cost of statistically monitoring the state of the process. With each sample drawn from the process a cost of measuring n units of the product (nB) is incurred. Where B represents the cost of inspecting a single unit of the product. A decision is made either to accept the process and do nothing, or reject and route the processed lot to screening and search for assignable causes at a cost rate W per occurrence. Since we are assuming a controlled process, the conditional probability of rejecting the process while in-control represents the type I error probability α. Thus, the average cost incurred per unit time of operation is:

$$E(C_m) = \frac{\mathrm{NB}}{\lambda}\left\{(1-\alpha)\frac{n}{N} + \alpha\left(1 + \frac{W}{\mathrm{NB}}\right)\right\}. \quad (6)$$

The expression above presents the relationship between the sampling design used and the process monitoring cost. Under a perfect scheme, a minimum unavoidable cost of (nB) per lot will be incurred. However, as α increase, the expected cost will increase up to the limiting value of $(\mathrm{NB} + W)$. It is important to note that equation (6) does not associate costs to the type II error. These costs are accounted for as part of the product inspection and deviation costs in the following sections.

4.2 Product inspection cost

Included in this element are the costs incurred due to inspecting lots of the product after processing. The inspection station receives these lots on an indication of the "no action" signal from the control chart. Under this assumption, Deming's (1986) approach for modeling the cost of inspection is utilized.. Two components are considered; the first represents initial cost of inspecting samples of n_s units plus the occasional cost of screening rejected lots, while the second represents the cost of further processing nonconforming units in accepted lots. An estimate of the later is based on the number of remaining units $(N - n_s)$, and the probability of acceptance Q. These reminders contain p nonconforming units, the processing of which represent a loss of A' per unit. This loss is equivalent to the value added on subsequent operations until the unit is detected. Consequently, the inspection cost per unit time can be estimated as (see Appendix, A1.2):

$$E(C_i) = \frac{NB}{\lambda(1-p)}\left\{1 + Q(1-p)\left(p\frac{A'}{B} - 1\right)\left(1 - \frac{n_s}{N}\right)\right\} \quad (7)$$

The expression above is in agreement with that given by Deming (1986) for estimating the total cost of in-coming material inspection. The function captures the relationship between the process conformance level and the economics of sampling. The term starting with Q in the equation is of special importance. According to Deming, at a nonconformance level $p > (B/A')$, lot screening is economically preferred from a direct cost standpoint. The quantity (B/A') was named by Deming the break-even quality. At this point, 100 percent inspection has no advantage over 0 percent, or no inspection. Where as, at a level of $p < (B/A')$, the same term in the equation will be negative, and inspection can be economically terminated. Whether a company chooses to adopt Deming's inspection criterion or follow traditional acceptance sampling practice, the expression above can be used to estimate the average inspection cost.

4.3 Cost of deviation

This cost element includes a direct cost of deviation from the product design target m, and an indirect cost of deviation from delivery schedule D. Let A represents the in-plant cost of disposing or reworking a nonconforming unit and ρ represents the time slack $(D - \lambda)$ after which delay costs of a per lot is charged. Utilizing the quadratic-loss function (Taguchi *et al.*, 1989), the expected total cost of deviation per unit time can be expressed as:

$$E(C_d) = \frac{1}{\lambda}\left\{\frac{\mathrm{NA}}{\Delta^2}\left[\sigma^2 + (\mu - m)^2\right] + \frac{a}{\rho^2}\nu^2\right\} \quad (8)$$

where υ^2 is the mean squared deviation from equation (5). Now, the expected total reactive cost of quality per unit time of operation can be obtained by adding equations (6)-(8).

5. Proactive cost of quality

The argument has been made that once process stability is attained, changes are required to improve the process. Hence, when the process is declared in-control, the options are either to maintain status quo or to improve performance. To this end, the cost of improvement can be defined as the costs incurred to attain a new level of the process quality. More specifically, we define improvement costs as the total costs incurred through:

- identifying new target values to better achieve customer expectations;
- reduce deviation between the current process average and the specified target; and
- reduce variation in the process output.

In this section we consider the incremental costs associated with a reduction ($\delta > 0$) of the process standard deviation σ or the deviation of its average μ from the target m. The costs involved are estimated using two components. The first accounts for the increase in operating costs due to test and evaluation of M proposed changes, while the second accounts for the cost of implementing an elected change. In modeling the first component, we assume that operating costs will increase by C dollar per unit time during test and evaluation stages. Further, we will associate an estimate of the process set up time with each change as well as an estimate of the time required to test and evaluate its effect. Estimates of both parameters can be expressed in terms of their expected values $\bar{t}$ and $\bar{\varepsilon}$ respectively. The former depends on the nature of the process and the ability to change its variables. Whereas, the later depends on the operating characteristics of the evaluation technique used.

On the other hand, implementation costs account for expenses incurred in making the elected change(s) part of the standard process. If all M changes are equally likely to result in the anticipated improvement, an estimate can be obtained in terms of their average value $\bar{Y}$. To account for the uncertainty of outcome and the possibility that additional trials may be required either to test new changes or to confirm the results, the expected total proactive cost is expressed in the form:

$$E(\mathrm{IC}) = \frac{1}{G}\{CM(\bar{t} + \bar{\varepsilon}) + \bar{Y}\} \tag{9}$$

where, G is a realization factor within the interval (0,1), representing the probability that the plan will lead to the anticipated improvement. In the ideal case, records on past improvement efforts can be used to estimate the value of G. With no relevant data, initial subjective estimates of G may be used and updated as records accumulate. It is common practice (Montgomery, 2001) to assign initial values of $G < 0.25$.

6. Illustrative application

A company manufactures steel forgings that are used on its assembly line. Design specifications on an important quality characteristic, slot width, are stated as 1.00 ± 0.006 inches. Owing to a high failure rate on assembly, the company followed traditional inspection practice as part of its quality assurance activities. Lots of 500 units are formed and submitted for inspection. An attribute-sampling plan is used with samples of 25 units and an acceptance number of one. Accepted lots are passed for further processing, while rejected lots are 100 percent inspected and nonconforming units are replaced.

A new quality engineer decides to construct a $\bar{x} - R$ control chart for the slot width. The charting scheme consists of drawing samples of five forgings, every 4 hours. Calculated sample statistics are plotted against three-sigma control limits. On achieving a state of statistical control, the slot width was shown to follow a normal distribution with mean 1.003 and standard deviation 0.002 inches. The cost of investigating chart signals is estimated as $1,000 per occurrence. The cost of inspection per unit is $0.5, while the in-plant cost of rework is $5.0 per unit. In addition, the cost of replacing a nonconforming forging after assembly is estimated as $50. It takes on the average 0.2 hours to process a unit, 0.05 hours to inspect it, and 0.2 hour to replace a nonconforming unit after assembly. The company faces delay costs of $5000 when lot completion time exceeds 124 hours due to interruptions of assembly operations. Other pertaining risk and time coefficients are obtained as follows:

$$\alpha \cong 1 - \prod_{1}^{2}(1 - \alpha_{\ell}) = 1 - (0.9973)^2 = 0.0054$$

where $\alpha_i = 2\int_{-\infty}^{-3}\phi(z)dz$, is the probability of type I error on each chart when a single out-of-control criterion is used. Under the normal distribution assumption, the proportion of nonconforming units is estimated as:

$$p = 1 - \int_{m-\Delta}^{m+\Delta} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu - x}{\sigma}\right)^2} dx = 0.067.$$

This results in a lot acceptance probability of:

$$Q = \sum_{y=0}^{1} \frac{25!}{y!(25-y)!}\hat{p}^{y}(1-\hat{p})^{25-y} = 0.495.$$

Since lot-screening time is larger than signal investigation time, equations (3) and (4) result in the following estimates:

$$E(d) = (e + \lambda_r p)\left[N - Q(1-\alpha)(N - n_s)\right] = 16.87$$

$$\nu^2 = (e + \lambda_r p)^2\left[N^2 - Q(1-\alpha)\left(N^2 - n_s^2\right)\right] = (22.6)^2.$$

Using equations (6)-(8) the expected reactive cost of quality $E(\mathrm{RC})_0$ is estimated at \$62.8 per hour at the current process level $\{L_0\}$.

The quality engineer has then decided to join the process team in their efforts to improve the process. Based on the information obtained form the control chart, the team concludes that action should be taken regarding the process average. Causing a shift of 1.5σ closer to the design target would reduce the fraction of nonconforming units to 0.003. At this new level $\{L_1\}$, reactive costs are estimated as $E(\mathrm{RC})_1=\$4.2$ per hour. This amounts to an estimated net savings of \$58.6 per hour of operation.

After examining possible cause-and-effect relationships, team members decided to test two improvement alternatives. The first involves a change in the part orientation during machining, while the second requires the use of a different cutting tool with a corresponding modification of the tool path. With a proper selection of cutting parameters, no change was expected in machining time. Team members estimate the setup time of the machine to be 0.5 hour, with implementation costs of \$500 and \$2000 for each alternative. Additional charges of \$200 per hour are anticipated due to an increase in the measurement and adjustment activities during operation. The quality engineer estimates that 4 hours will be required to evaluate each alternative. Assuming a realization factor of 0.25, to allow for verification, the expected cost of improvement $E(\mathrm{IC})$ is obtained using equation (9) as \$17,200.

To study the economics of such an investment, the engineer decides to use equation (2) to study the net present worth of the improvement. On contacting the accounting department, the company's effective rate of return is found to be 3 percent per month (this is overstated to reflect the risk involved). The engineer also knows that the total operating time is 120 hours per month, and that production is expected to continue over the next 12 months. Based on this information, equation (2) resulted in a net present worth of \$52796, which satisfies the economic criterion for process improvement.

7. Model performance

In this section, we utilize the previous application to demonstrate the effect of the various input variables on the net present worth function. In particular, we examine the effect of the factors involved in determining the proactive cost, while holding the incremental savings at a constant level. This is equivalent to investigating the effect of the six factors used in estimating the cost of improvement given by equation (9). The statistical design used is an un-replicated 2^6 factorial. Each factor is assigned two levels as shown in Table I. These levels were selected based on the information provided by the process improvement team. It should be pointed out that results of the forthcoming analysis are expected to vary should these values change.

The design matrix used and values of the NPW at the corresponding levels of these parameters are shown in Table II. The procedure used follows closely that given by Montgomery (2001) for un-replicated two-level factorial designs. A half-normal plot of the estimated effects is shown in Figure 5. The plot indicates that some of the interactions involving two or more factors can be considered as residuals with 44 degrees of freedom. Results from an analysis of variance (ANOVA) for the selected variables and interactions are shown in Table III.

All selected effects are shown to be relatively important. Interpretations of significant interactions have led to the following observations:

- The average evaluation time $\bar{\varepsilon}$ has greater effect on the NPW at high levels of the incremental cost of operation C. The effect tens to increase as the realization factor G, and number of changes M approach their high levels. This indicates the need for statistical procedures with high power for evaluating process changes in the early stages of process improvement.
- Changes in the realization factor G tend to have higher effect on the NPW at the high level of the average implementation cost $\bar{Y}$. In other words, one should pay special attention while estimating values of realization when the average implementation cost is high.
- Changes in the incremental cost of operation C are shown to have higher effect on the *NPW* at the high level of $\bar{t}$ the average setup time. This is a reinforcement of the fact that setup and implementation need to be addressed jointly in planning and evaluating process improvement efforts.

8. Summary and conclusions

In this paper, we propose a mathematical model for the cost of quality, which accounts for the value of process improvement. The model is developed to include two major cost elements. One is termed reactive cost, which accounts for quality related costs incurred at a given stable level of operation. These include the process monitoring cost; product inspection cost and the loss due to deviation form the part design target and delivery schedule. The second element considers the cost of attaining an improved level of conformance and hence is termed proactive cost. This element accounts for the cost of

Table I.
Input factors and levels used in the experiment

Factor	Level Low (−1)	High (+1)
$\bar{t}$	0.25	1.25
$\bar{Y}$	1000	5,000
C	100	500
$\bar{\varepsilon}$	2	10
G	0.1	0.5
M	1	5

Run	$\bar{t}$	$\bar{Y}$	C	$\bar{\varepsilon}$	G	M	NPW
1	−1	−1	−1	−1	−1	−1	57,747
2	1	−1	−1	−1	−1	−1	56,747
3	−1	1	−1	−1	−1	−1	17,747
4	1	1	−1	−1	−1	−1	16,747
5	−1	−1	1	−1	−1	−1	48,747
6	1	−1	1	−1	−1	−1	43,747
7	−1	1	1	−1	−1	−1	8,747
8	1	1	1	−1	−1	−1	3,747
9	−1	−1	−1	1	−1	−1	49,747
10	1	−1	−1	1	−1	−1	48,747
11	−1	1	−1	1	−1	−1	9,747
12	1	1	−1	1	−1	−1	8,747
13	−1	−1	1	1	−1	−1	8,747
14	1	−1	1	1	−1	−1	3,747
15	−1	1	1	1	−1	−1	−31,253
16	1	1	1	1	−1	−1	−36,253
17	−1	−1	−1	−1	1	−1	67,547
18	1	−1	−1	−1	1	−1	67,347
19	−1	1	−1	−1	1	−1	59,547
20	1	1	−1	−1	1	−1	59,347
21	−1	−1	1	−1	1	−1	65,747
22	1	−1	1	−1	1	−1	64,747
23	−1	1	1	−1	1	−1	57,747
24	1	1	1	−1	1	−1	56,747
25	−1	−1	−1	1	1	−1	65,947
26	1	−1	−1	1	1	−1	65,747
27	−1	1	−1	1	1	−1	57,947
28	1	1	−1	1	1	−1	57,747
29	−1	−1	1	1	1	−1	57,747
30	1	−1	1	1	1	−1	56,747
31	−1	1	1	1	1	−1	49,747
32	1	1	1	1	1	−1	48,747
33	−1	−1	−1	−1	−1	1	48,747
34	1	−1	−1	−1	−1	1	43,747
35	−1	1	−1	−1	−1	1	8,747
36	1	1	−1	−1	−1	1	3,747
37	−1	−1	1	−1	−1	1	3,747
38	1	−1	1	−1	−1	1	−21,253
39	−1	1	1	−1	−1	1	−36,253
40	1	1	1	−1	−1	1	−61,253
41	−1	−1	−1	1	−1	1	8,747
42	1	−1	−1	1	−1	1	3,747
43	−1	1	−1	1	−1	1	−31,253
44	1	1	−1	1	−1	1	−36,253
45	−1	−1	1	1	−1	1	−196,253
46	1	−1	1	1	−1	1	−221,253
47	−1	1	1	1	−1	1	−236,253

(continued)

Table II. Design matrix and calculated NPW values

Run	$\bar{t}$	$\bar{Y}$	C	$\bar{\varepsilon}$	G	M	NPW
48	1	1	1	1	−1	1	−261,253
49	−1	−1	−1	−1	1	1	65,747
50	1	−1	−1	−1	1	1	64,747
51	−1	1	−1	−1	1	1	57,747
52	1	1	−1	−1	1	1	56,747
53	−1	−1	1	−1	1	1	56,747
54	1	−1	1	−1	1	1	51,747
55	−1	1	1	−1	1	1	48,747
56	1	1	1	−1	1	1	43,747
57	−1	−1	−1	1	1	1	57,747
58	1	−1	−1	1	1	1	56,747
59	−1	1	−1	1	1	1	49,747
60	1	1	−1	1	1	1	48,747
61	−1	−1	1	1	1	1	16,747
62	1	−1	1	1	1	1	11,747
63	−1	1	1	1	1	1	8,747
64	1	1	1	1	1	1	3,747

Table II.

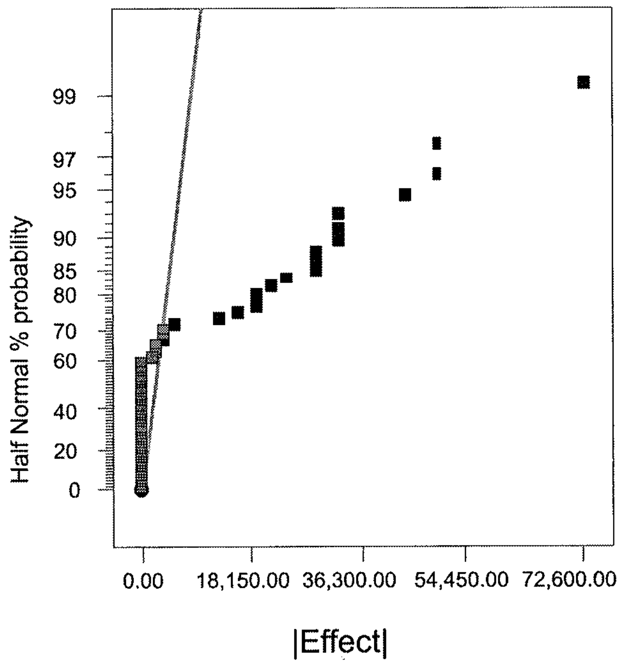

Figure 5.
Half normal plot of estimated effects

Source	Sum of squares	DF	Mean square	F value	Prob > F
Model	3.217E+011	19	1.693E+010	1,017.66	< 0.0001
$\bar{t}$	4.666E+008	1	4.666E+008	28.04	< 0.0001
$\bar{Y}$	9.216E+009	1	9.216E+009	553.85	< 0.0001
C	3.779E+010	1	3.779E+010	2,271.12	< 0.0001
$\bar{\varepsilon}$	2.986E+010	1	2.986E+010	1,794.46	< 0.0001
G	8.433E+010	1	8.433E+010	5,068.04	< 0.0001
M	3.779E+010	1	3.779E+010	2,271.12	< 0.0001
$\bar{t}C$	2.074E+008	1	2.074E+008	12.46	0.0010
$\bar{Y}G$	4.096E+009	1	4.096E+009	246.15	< 0.0001
$C\bar{\varepsilon}$	1.327E+010	1	1.327E+010	797.54	< 0.0001
CG	1.680E+010	1	1.680E+010	1,009.38	< 0.0001
CM	1.680E+010	1	1.680E+010	1,009.38	< 0.0001
$\bar{\varepsilon}G$	1.327E+010	1	1.327E+010	797.54	< 0.0001
$\bar{\varepsilon}M$	1.327E+010	1	1.327E+010	797.54	< 0.0001
GM	1.680E+010	1	1.680E+010	1,009.38	< 0.0001
$C\bar{\varepsilon}G$	5.898E+009	1	5.898E+009	354.46	< 0.0001
$C\bar{\varepsilon}G$	5.898E+009	1	5.898E+009	354.46	< 0.0001
CGM	7.465E+009	1	7.465E+009	448.62	< 0.0001
$\bar{\varepsilon}GM$	5.898E+009	1	5.898E+009	354.46	< 0.0001
$C\bar{\varepsilon}GM$	2.621E+009	1	2.621E+009	157.54	< 0.0001
Residual	7.322E+008	44	1.664E+007		
Cor total	3.225E+011	63			

Table III. ANOVA for the NPW (partial sum of squares)

introducing planned changes to the process as part of continuing efforts to improve its conformance. Using incremental economics the two cost functions are assembled to obtain the net worth of such improvement. To ensure that higher conformance should cost less, an economic criterion is used to restrict the net worth of improvement within positive boundaries.

The proposed model provides separate representations of typical cost centers at the process level. This would enable practitioners to analyze the various costs and their relationships to pertaining process parameters. Also, the model is modular in nature and can be easily reduced or expanded to suite specific applications. For example, in applications where sampling inspection is not utilized, the model can be reduced by modifying values of n_s, P and Q in equations (4), (5) and (7). Similarly, in applications where other conformance targets are mandated (e.g. environmental), additional quadratic terms can be included in equation (8) to reflect this. Furthermore, The proposed model can be used to evaluate improvement projects aimed at increasing the production rate, decreasing the process setup time, or lot size. Finally, while the proposed model allows a micro-level examination of the economics of process improvement, it can be modified to incorporate revenue-side quality benefits. These may include increased market share and profit margins, which are usually left out in traditional cost-oriented models. Due considerations of such gains are essential in future macro-level applications of the proposed model. The authors are currently researching these as well as applications involving the economic selection of control chart design parameters.

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Appendix. Mathematical derivations

A.1 Expected delay and mean squared deviation

Under the assumptions made in Section 3, the lot processing time λ is assumed constant and due to the independence between the control chart action limits and those of the product acceptance plan, the probability distribution function of the lot completion time λ_t can be expressed as:

$$\Psi(\lambda_t) = \begin{cases} (1-\alpha)Q & \lambda_t = \lambda + \lambda_s \\ (1-\alpha)P & \lambda_t = \lambda + \lambda_{sc} \\ \alpha & \lambda_t = \lambda + \lambda_{sc}\theta + \lambda_v(1-\theta) \end{cases} \quad \text{(A.1.1)}$$

Where λ_v = the time to investigate a chart signal, $\theta = 1$ if $\lambda_{sc} \geq \lambda_v$, and $\theta = 0$ otherwise.

Hence, the expected cycle time is given by:

$$E(\lambda_t) = \lambda + (1-\alpha)[\lambda_s + P(\lambda_{sc} - \lambda_s)] + \alpha[\lambda_{sc}\theta + \lambda_v(1-\theta)]. \quad \text{(A.1.2)}$$

That is, the average delay that would be experienced in having N units available is:

$$E(d) = (1-\alpha)[\lambda_s + P(\lambda_{sc} - \lambda_s)] + \alpha[\lambda_{sc}\theta + \lambda_v(1-\theta)]. \quad \text{(A.1.3)}$$

For the case where, $\lambda_{sc} \geq \lambda_v$, the indicator variable $\theta = 1$, and the equation (A.1.3) results in:

$$E(d) = (e + \lambda_r p)\left[N - Q(1-\alpha)(N - n_s)\right]. \quad \text{(A.1.4)}$$

Similarly, for $\theta = 0$, the average delay is given by:

$$E(d) = (e + \lambda_r p)(1-\alpha)\left[N - Q(N - n_s)\right] + \alpha\lambda_v. \quad \text{(A.1.5)}$$

The mean squared deviation from a target value of zero is obtained as:

$$\nu^2 = E(d-0)^2 = E\left(d^2\right). \quad \text{(A.1.6)}$$

Based in equation (A.1.4) where $\theta = 1$:

$$\nu^2 = (e + \lambda_r p)^2\left[N^2 - Q(1-\alpha)\left(N^2 - n_s^2\right)\right]. \quad \text{(A.1.7)}$$

For the case where $\theta = 0$, equation (A.1.5) results in:

$$\nu^2 = (e + \lambda_r p)^2(1-\alpha)\left[N^2 - Q\left(N^2 - n_s^2\right)\right] + \alpha\lambda_v^2. \quad \text{(A.1.8)}$$

A.2 Expected inspection costs

The two cost components considered are the initial cost of inspection (C_1), and downstream cost (C_2). The first represents initial cost of inspecting samples of n_s units plus the occasional cost of screening rejected lots. The assumption of constant lot size requires the replacement of any nonconforming unit found. Since these replacements have a proportion p nonconforming units, it will take on the average $1/(1-p)$ units to find a conforming one. Consequently, the costs involved are:

$$E(C_1) = \frac{\text{NB}}{(1-p)}\left\{P + Q\frac{n_s}{N}\right\}. \quad \text{(A.2.1)}$$

Downstream costs represent those of further processing nonconforming units in accepted lots.

An estimate of such cost is based on the number of remaining units ($N - n_s$), and the probability of acceptance Q. These remainders contain p nonconforming units, the processing of which represent a loss of A'per unit. The replacement cost is $B/(1-p)$ as before. Hence, the downstream cost can be estimated as:

$$E(C_2) = \frac{\mathrm{NB}}{(1-p)}\left\{\left(1-\frac{n_s}{N}\right)Qp\left[\frac{A'}{B}(1-p)+1\right]\right\}. \quad \text{(A.2.2)}$$

Equation (7) is obtained by multiplying the sum of equations (A.2.1) and (A.2.2) by $1/\lambda$ (the reciprocal of the lot processing time).